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SHORT NOTE

Strain from three stretches—a simple Mohr circle solution

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Abstract—The estimation of the strain ellipse from data consisting of longitudinal strains measured along different directions is one of the basic problems of strain analysis and one which has recently received renewed attention. A graphical method based on the Mohr circle, described more than 40 years ago by Glenn Murphy but overlooked by geologists, represents the simplest solution of the problem yet devised.

INTRODUCTION

Two widely-used graphical methods for evaluating the strain ellipse from the longitudinal strains measured along three or more differently oriented lines (a so-called strain rosette), are described by Ramsay & Huber (1983, p. 91). Both are based on the Mohr circle but involve either auxiliary geometrical constructions or the construction of separate overlays to the Mohr diagram. Though recent algebraic solutions to the problem (Sanderson 1977, Ragan 1987, De Paor 1988) are less time-consuming to obtain, they do not allow the direct visual assessment of the quality of the data that is possible when the graphical methods are employed. This note is intended to draw the attention of geologists to an extremely simple and direct graphical method devised more than 40 years ago to solve the strain rosette problem.

CONSTRUCTION

The construction described by the American engineer, Glenn Murphy, to analyse infinitesimal strain rosettes (Murphy 1945) can be readily adapted for finite-strain analysis. Figure 1(a) shows an example of data to be analysed; a rosette formed from the stretched belemnites featured in Ramsay & Huber (1983, pp. 94 and 104). For ease of discussion, let us label the three lines of the rosette A, B and C according to their calculated longitudinal strain magnitude, such that $\lambda'_A > \lambda'_B > \lambda'_C$.

The Mohr circle is constructed by first drawing the γ' (vertical) axis of the Mohr diagram and, after adopting a convenient scale, drawing three vertical lines, $\lambda' = \lambda'_A$, $\lambda' = \lambda'_B$ and $\lambda' = \lambda'_C$ (Fig. 1b). At an arbitrary point, P, on the vertical line $\lambda' = \lambda'_B$ we draw the strain rosette so that arm B is coincident with the line $\lambda' = \lambda'_B$; arms A and C intersect the lines $\lambda' = \lambda'_A$ and $\lambda' = \lambda'_C$ at points A and C, respectively (Fig. 1b). The perpendicular bisectors to the lines AP and CP intersect at a point which is the centre of the Mohr circle. The constructed circle passes through points P, A and C. The λ' axis of the Mohr diagram can now be drawn through the circle's centre. The principal strains magnitudes (λ'_1 and λ'_2) are given as usual by the position of points where the Mohr circle cuts the λ' axis of the diagram (Fig. 1b). The orientations of the principal strain axes, in the reference frame of the rosette in Fig. 1(b), are given by the lines joining P to λ'_1 and λ'_2, respectively.

A proof of the construction can be found in Murphy (1945). The basis of the construction can be readily appreciated once it is realized that point P at the centre of the rosette also functions at the *pole* of the Mohr circle; a point about which directions in real space can be related to their representation in Mohr space (Mandl & Shippam 1981, Means 1982, Cutler & Elliott 1983, Allison 1984). Certain elements of the proposed method are contained in procedures described by Ragan (1973) but the use of the pole here makes the method more readily understandable by bringing the geometry of the strained objects and of the construction into closer correspondence.

It should be noted that the constructed Mohr circle is

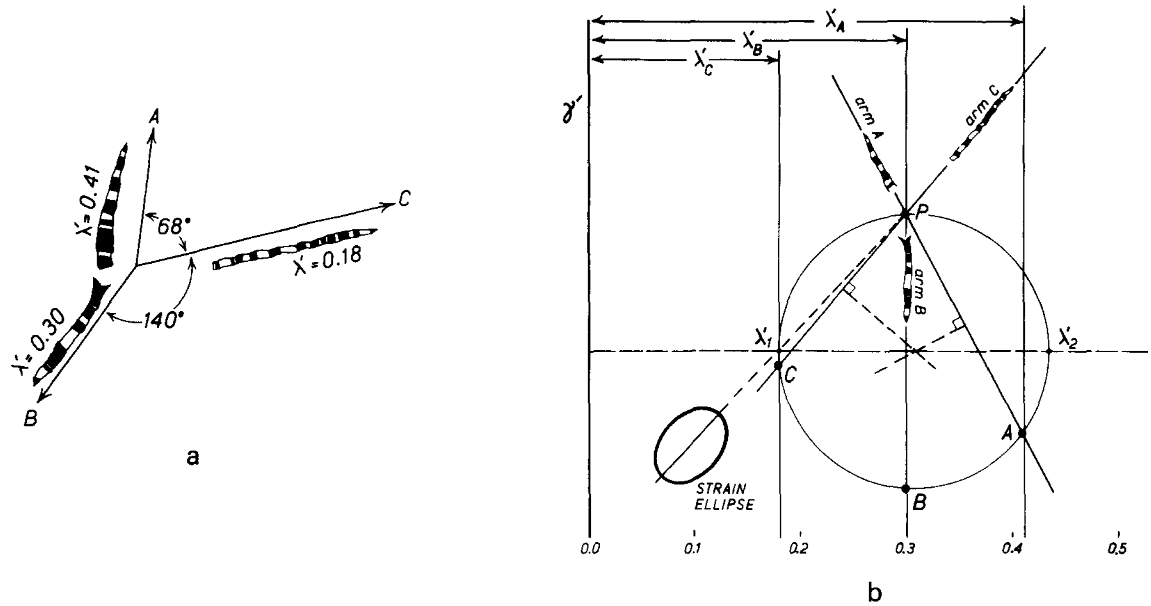

Fig. 1. (a) The data (Ramsay & Huber 1983): three longitudinal strains arranged in a rosette, $\lambda'_A > \lambda'_B > \lambda'_C$. (b) The construction: draw γ' axis of Mohr diagram and verticals representing the three λ' values. Rosette is superimposed with arm B coincident with vertical line $\lambda' = \lambda'_B$. Points A and C are located where arms A and C of rosette intersect verticals $\lambda' = \lambda'_A$ and $\lambda' = \lambda'_C$, respectively. Finally, Mohr circle is drawn through P, A and C.

upside down with respect to that resulting from the Ramsay & Huber solution (1983, p. 104). In fact, as pointed out by Treagus (1987) the use of the pole properly implies a γ' clockwise-up convention (see De Paor & Means 1984 and Treagus 1987 for discussion of Mohr sign conventions).

CONCLUSIONS

The three stretch problem can be considered a classic one in the field of strain analysis and is an exercise which forms part of many undergraduate structural courses. The potential total saving of practical class time justifies the application of Murphy's ingenious, but previously overlooked, method.

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